

Bandwidth Efficient Coding for Satellite Communications*

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RESEARCH PURPOSE

The **purpose** of this research is to devise an error control coding scheme to achieve **large coding gain** and **high reliability** by using coded modulation with **reduced** decoding complexity.

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CODED MODULATION ALONE

- To achieve a 3 to 5 dB coding gain and moderate reliability, the decoding complexity is quite modest.
- In fact, to achieve a 3 dB coding gain, the decoding complexity is quite simple, no matter whether **trellis coded modulation (TCM)** or **block coded modulation (BCM)** is used.
- However, to achieve coding gains exceeding 5 dB, the decoding complexity increases **drastically**, and the implementation of the decoder becomes very expensive and unpractical.

A BASIC QUESTION

- How can we achieve large coding gains and high reliability by using coded modulation with reduced decoding complexity ?

AN ANSWER

- Use coded modulation in conjunction with concatenated (or cascaded) coding.
- A good short bandwidth efficient modulation code (trellis or block) is used as the inner code and relatively powerful Reed–Solomon (RS) code is used as the outer code.
- With properly chosen inner and outer codes, a concatenated coded modulation scheme not only can achieve large coding gains and high reliability with good bandwidth efficiency but also can be practically implemented
- This combination of coded modulation and concatenated coding really offers a way of achieving the best of three worlds, reliability and coding gain, bandwidth efficiency and decoding complexity.

PROPOSED SCHEME

- A concatenated (or cascaded) coded modulation scheme.
- For **NASA** high-speed satellite communications for large data file transfer where very high reliability is required.
- The outer code C_2 is an (n_2, k_2) RS code with symbols from $\text{GF}(2^b)$.
- The outer code is **interleaved** to a **depth** of m .
- The inner code is a bandwidth efficient **block M-ary PSK** code of length n_1 and dimension
$$k_1 = mb.$$
- Under the same research project, we have investigated concatenated coding with TCM inner codes.

THE OVERALL CONCATENATED CODED MODULATION SCHEME

- The outer code C_2 is an (n_2, k_2) RS code over $GF(2^b)$, which is designed to correct t_2 or fewer symbol errors with $0 \leq t_2 \leq \lfloor (n_2 - k_2)/2 \rfloor$.

- The inner code C_1 is a 2^l -PSK code of length n_1 and dimension

$$k_1 = \sum_{i=1}^l k_{b,i}$$

where $k_1 = mb$.

- The outer code C_2 is interleaved to a depth of m .
- The encoding consists of two stages, the outer and inner encodings.
- The decoding consists of two stages, the inner and outer decodings.
- When the receiver fails to decode a received block, the block is erased and the receiver raises a **flag**.
- In the event of an erasure, we could either request a retransmission or accept the erroneous block with alarm.

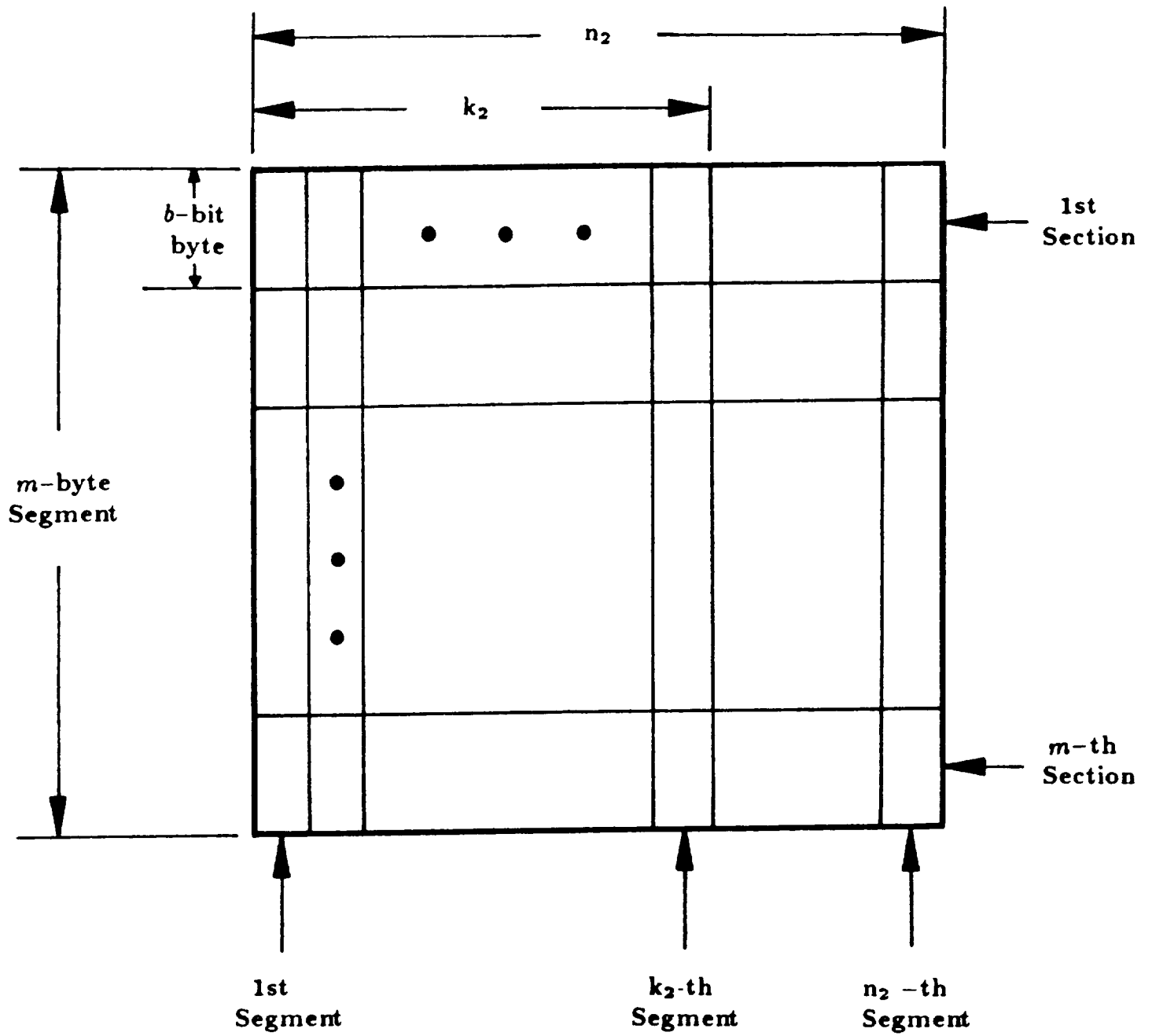


Figure 1 A Segment-Array

ERROR PERFORMANCE OF THE OVERALL SCHEME

- Let P_c , $P_{e,}$ and $P_{e,}$ be the probabilities of a correct decoding, an erasure and an incorrect decoding for an entire received code block respectively.
- Lower bound on P_c and upper bounds on $P_{e,}$ and $P_{e,}$ have been derived for an AWGN channel.
- Let \check{P}_c denote a lower bound on P_c .
- Then $1-\check{P}_c$ is an upper bound on the **total probability** of a decoding failure and a decoding error.
- Let $\hat{P}_{e,}$ denote an upper bound on $P_{e,}$.
- The performance of the proposed concatenated coded modulation scheme is measured by the pair, $\hat{P}_{e,}$ and $1-\check{P}_c$.
- We can compute the coding gains of the proposed scheme over the uncoded QPSK modulation system either in terms of decoded **block-error rates** or in terms of decoded **bit-error rates**.

- For data file transfer, the block-error rates should be used as the measure of the error performance of the scheme.
- There are two types of bit-error rates, denoted $P_{b,1}$ and $P_{b,2}$.
- $P_{b,1}$ is computed based on the block error probability $P_{e,r}$ using the approximation,

$$P_{b,1} = (d_r / 2n_r) \cdot P_{e,r}$$

- $P_{b,1}$ is a measure of bit-error performance of the proposed scheme when retransmission is allowed.
- $P_{b,2}$ is computed based on the total probability $1 - P_e$ of a decoding failure and a decoding error of a code block using the approximation,

$$P_{b,2} = (d_r / 2n_r) \cdot (1 - P_e).$$

- $P_{b,2}$ is used as the measure of bit-error performance of the scheme when retransmission is not available or allowed.

TWO SPECIFIC CONCATENATED CODED MODULATION SCHEMES

SCHEME – I

- The outer code C_2 is the NASA standard (255,223) RS code over $GF(2^8)$ which has minimum distance 33. It is used to correct up to 16 symbol errors.
- The inner code C_1 is an 8-PSK code with $n_1 = 8$, $k_1 = 16$, $D[C_1] = 4$, $R[C_1] = 1$ and $\gamma[C_1] = 3$ dB (over uncoded QPSK).
- The outer code is interleaved to a depth of $m = 2$.
- The overall effective rate of the scheme is

$$R_{eff} = (k_2 / n_2) \cdot R[C_1] = 0.875.$$

- The inner code has a 4-state trellis structure and can be decoded with a soft-decision Viterbi decoder.

ERROR PERFORMANCE

- With $\text{SNR} = 9 \text{ dB/symbol}$ ($6.57 \text{ dB/infor. bit}$),

$$P_{e,r} \leq 6.28 \times 10^{-26}$$

$$1 - P_e \leq 4.95 \times 10^{-16}$$

- With $\text{SNR} = 10 \text{ dB/symbol}$ ($5.57 \text{ dB/infor. bit}$),

$$P_{e,r} \leq 6.80 \times 10^{-41}$$

and $1 - P_e$ is very small.

CODING GAIN OVER QPSK

- At the block-error rate $= 10^{-7}$,

$$G_b = 8 \text{ dB/symbol.}$$

- At the block-error rate $= 10^{-10}$,

$$G_b = 9 \text{ dB/symbol.}$$

- At the bit-error rate $P_{b,1} = 10^{-12}$,

$$G_{b,1} = 9.80 \text{ dB/symbol (9.20 dB/infor. bit).}$$

The required SNR to achieve $P_{b,1} = 10^{-12}$ is 7.10 dB/symbol (4.60 dB/infor. bit).

- At the bit-error rate $P_{b,2} = 10^{-6}$,

$$G_{b,2} = 5.52 \text{ dB/symbol (4.94 dB/infor. bit)}.$$

The required SNR to achieve $P_{b,2} = 10^{-6}$ is 8.04 dB/symbol (5.61 dB/infor. bit).

- At the bit-error rate $P_{b,2} = 10^{-10}$,

$$G_{b,2} = 7.60 \text{ dB/symbol (7.02 dB/infor. bit)}.$$

The required SNR to achieve $P_{b,2} = 10^{-10}$ is 8.50 dB/symbol (6.07 dB/infor. bit).

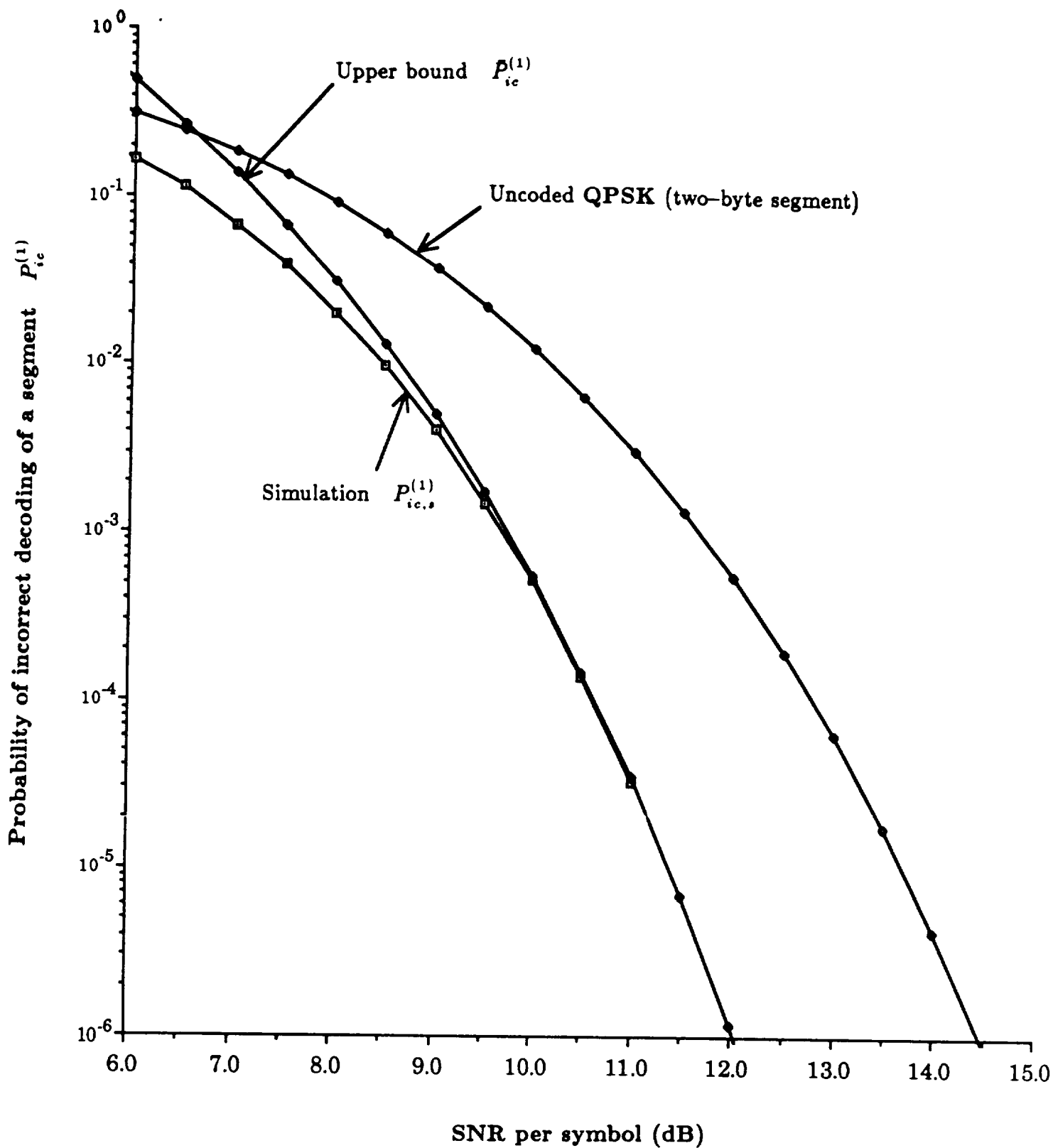


Figure 2 Error performance of the 4-state 8-PSK block code (the 4-th code in Table 1)

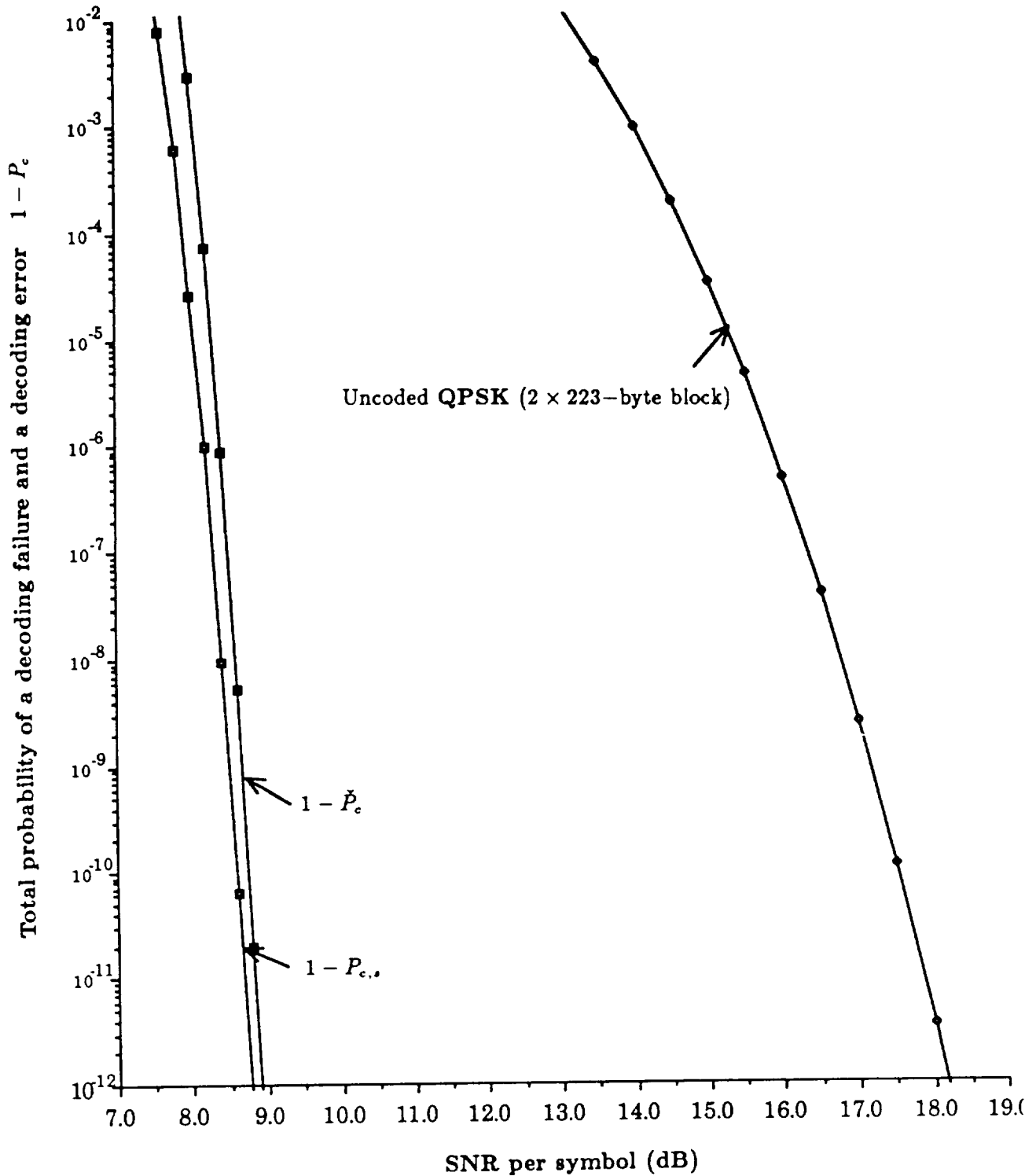


Figure 3 The total probability of a decoding failure and a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 4-state 8-PSK block inner code (the 4-th code in Table 1)

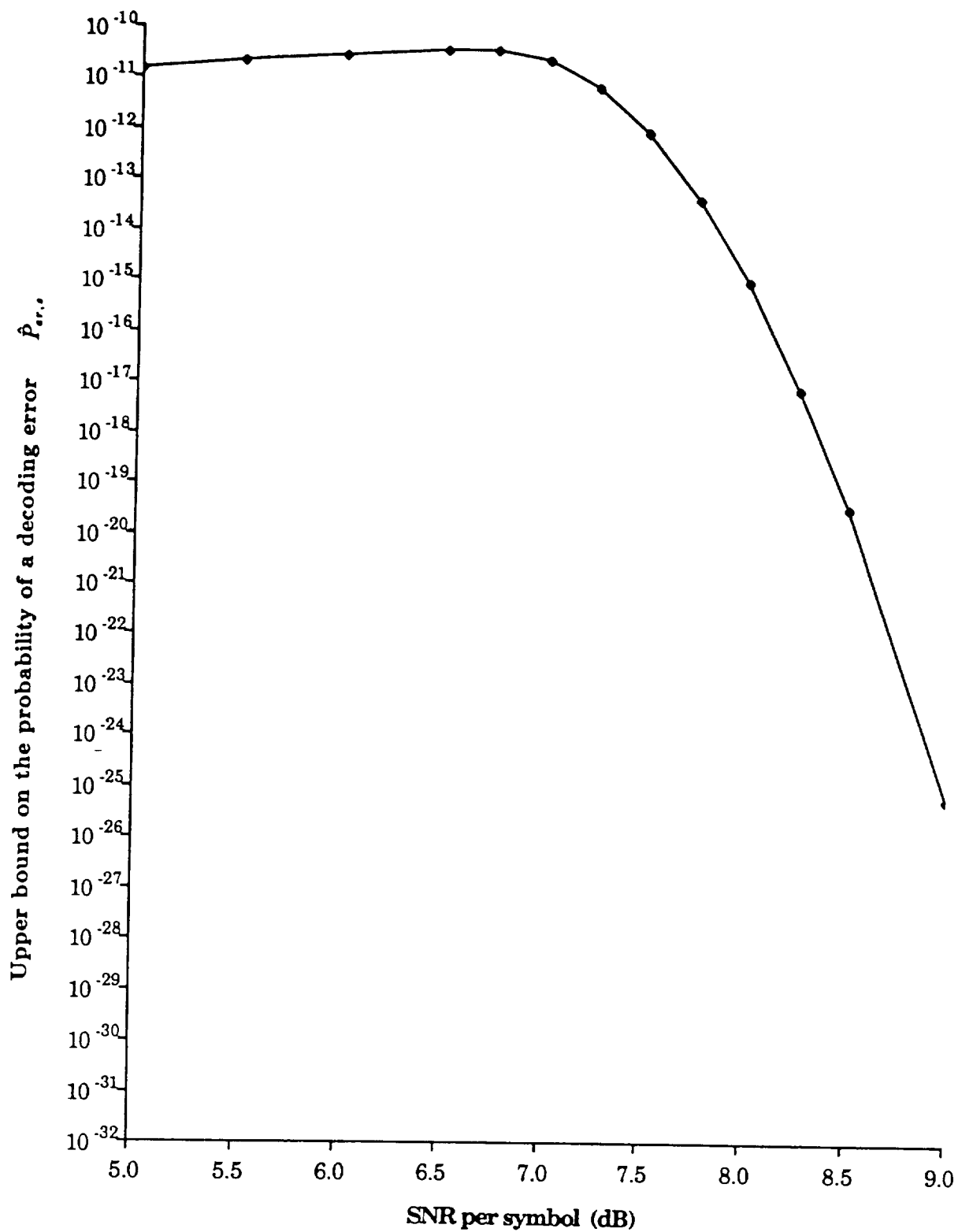


Figure 4 The probability of a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 4-state 8-PSK block inner code (the 4-th code in Table 1)

SCHEME – II

- The outer code is the NASA standard (255,223) RS code over $\text{GF}(2^8)$.
- The inner code C_1 is an 8-PSK code of length 16 and dimension $k_1 = 36$ with $D[C_1] = 4$, $R[C_1] = 9/8$ and $\gamma[C_1] = 3.52$ dB (over uncoded QPSK).
- The outer code is interleaved to a depth of $m = 9$.
- The overall effective rate of the scheme is

$$R_{eff} = (223/255) \cdot (9/8) = 0.9838.$$

- The inner code has a 16-state trellis diagram which consists of two identical parallel 8-state trellis sub-diagrams with no cross connection between them.
- The probability of an incorrect decoding for this code is

$$\begin{aligned}
P_{ic}^{(1)} \leq & 248 \operatorname{erfc}(\sqrt{\rho}) + 1920 \operatorname{erfc}(\sqrt{2(2-\sqrt{2})\rho}) + 30720 \operatorname{erfc}\left(\frac{\sqrt{2(9-4\sqrt{2})\rho}}{2}\right) \\
& + 15360 \operatorname{erfc}\left(\frac{\sqrt{2(8-3\sqrt{2})\rho}}{2}\right) + 16384 \operatorname{erfc}(2\sqrt{(2-\sqrt{2})\rho}) \\
& + 245760 \operatorname{erfc}\left(\frac{\sqrt{3(8-3\sqrt{2})\rho}}{2}\right) + 262144 \operatorname{erfc}\left(\frac{\sqrt{2(16-7\sqrt{2})\rho}}{2}\right) .
\end{aligned}$$

- At the 10^{-6} decoded block error rate, this inner code provides a 2.20 dB real coding gain over the uncoded QPSK.

ERROR PERFORMANCE

- With $\text{SNR} = 10 \text{ dB/symbol}$ (or $7.06 \text{ dB/infor. bit}$),

$$P_{e,r} \leq 6.91 \times 10^{-41}$$

$$1 - P_e \leq 2.08 \times 10^{-12}$$

CODING GAIN

- At the block-error rate $= 10^{-7}$,

$$G_B = 7 \text{ dB/symbol.}$$

- At the block-error rate $= 10^{-10}$,

$$G_B = 8 \text{ dB/symbol.}$$

- At the bit-error rate $P_{b_1} = 10^{-31}$,

$$G_{b_1} = 15 \text{ dB/symbol.}$$

- At the bit-error rate $P_{b_2} = 10^{-10}$,

$$G_{b_2} = 6.26 \text{ dB/symbol (6.19 dB/infor. bit).}$$

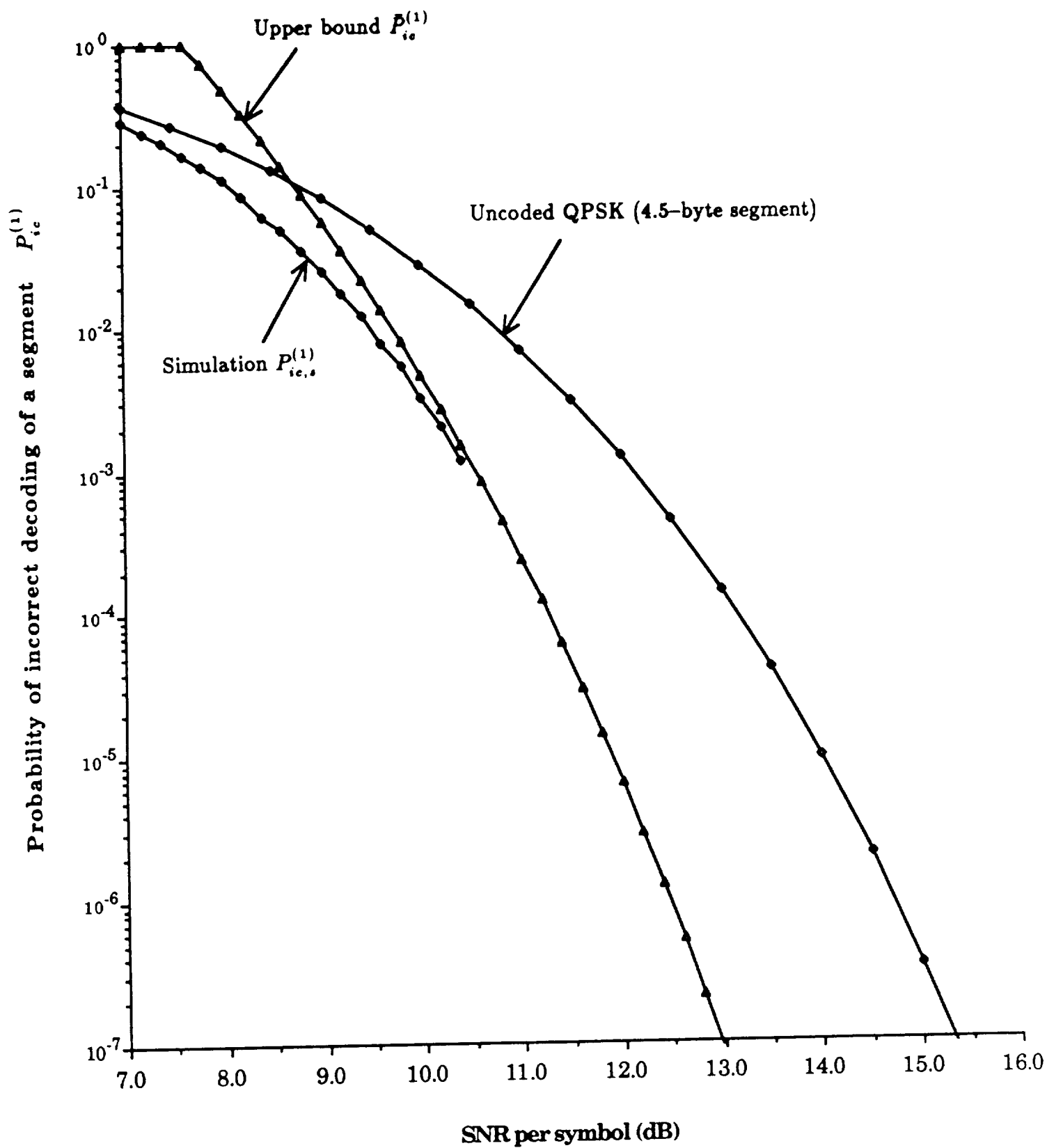


Figure 5 Error performance of the 16-state 8-PSK block inner code (the 5-th code in Table 1)

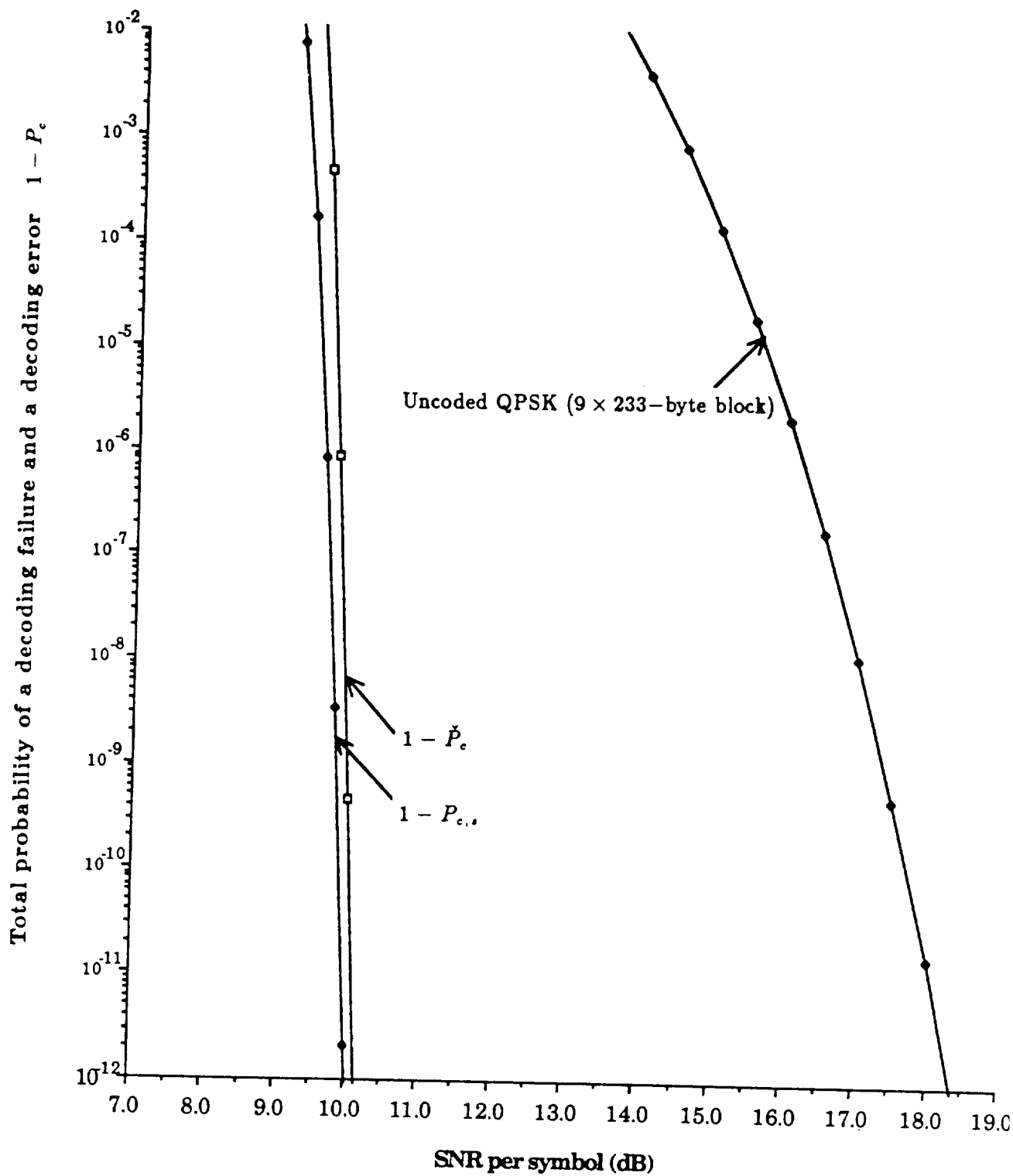


Figure 6 The total probability of a decoding failure and a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 16-state 8-PSK block inner code (the 5-th code in Table 1)

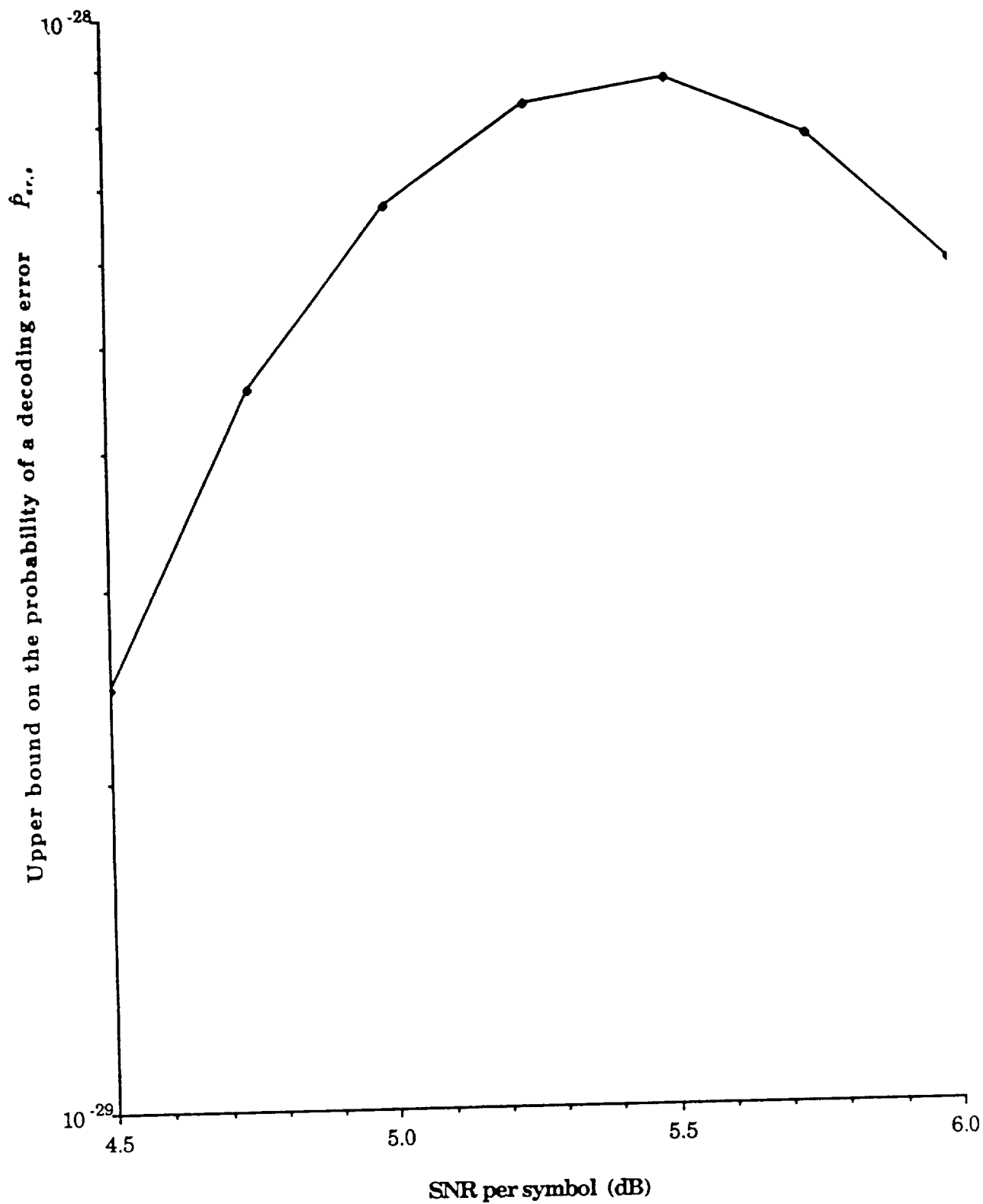


Figure 7 The probability of a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 16-state 8-PSK block inner code (the 5-th code in Table 1)

REMARK

- The inner code decoder can be implemented to perform both decoding and **erasure** operations.
- In this case, the outer code decoder is devised to correct both symbol errors and erasures.

SESSION VI

TECHNOLOGY NEEDS/OPPORTUNITIES FOR FUTURE MISSIONS

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